Generating Complex Baseband and Analytic Bandpass Signals

by Richard Lyons, Nov. 2011

There are so many different time- and frequency-domain methods for generating complex baseband and analytic bandpass signals that I had trouble keeping those techniques straight in my mind. Thus, for my own benefit, I created a kind of *reference* table showing those methods. I present that table for your viewing pleasure in this blog.

For clarity, I define a complex baseband signal as follows: derived from an input analog $x_{bp}(t)$bandpass signal whose spectrum is shown in Figure 1(a), or discrete input $x_{bp}(n)$ bandpass signal whose spectrum is shown in Figure 1(b), a complex baseband signal is an $x_{BB}(n)$ sequence whose spectrum is that shown in Figure 1(c). The sample rate of an $x_{bp}(n)$ input sequence is defined as f_s Hz.

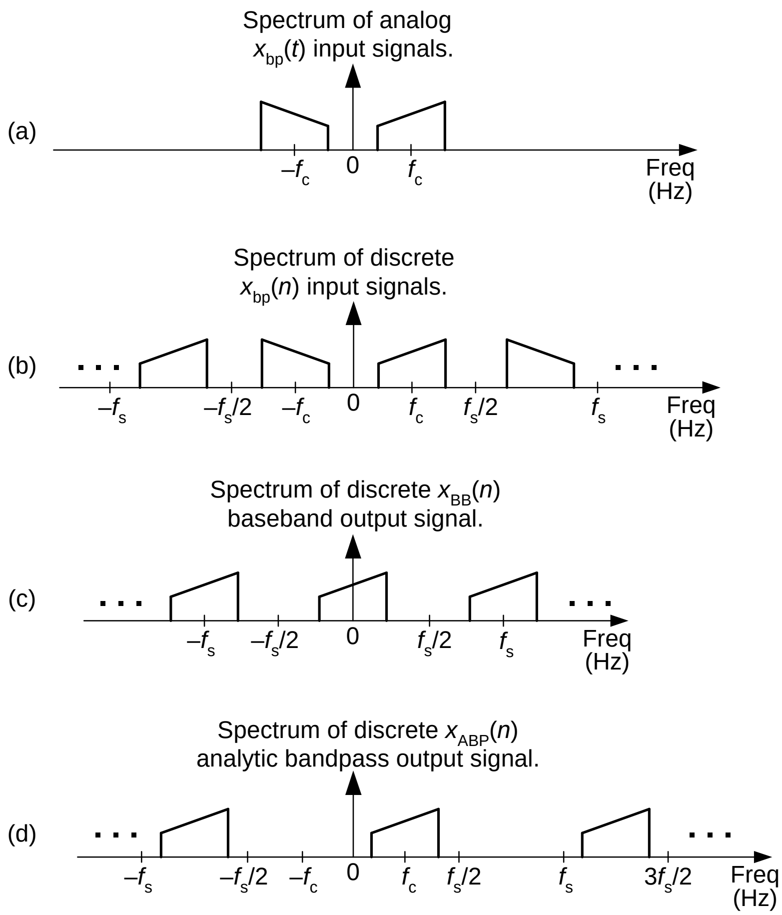


Figure 1.

Based on the same analog $x_{bp}(t)$ or discrete $x_{bp}(n)$ input bandpass signal, an analytic bandpass signal is an $x_{ABP}(n)$ sequence whose spectrum is that shown in Figure 1(d).

I realize that, by strict definition, an analytic signal has no negative-frequency spectral energy. And because our $x_{ABP}(n)$ output bandpass signal is a discrete sequence it has spectral replications in its negative-frequency spectral region——so calling $x_{ABP}(n)$ an analytic signal seems incorrect. We'll bypass that controversy by saying that a discrete sequence is *analytic* if it has no spectral energy in the frequency range of $-f_s/2$ to zero Hz.

Table 1, below, presents my Hit Parade of complex baseband and analytic bandpass signal generation methods. In that table "LPF" means a lowpass, linear-phase, tapped-delay line, FIR filter. All discrete Fourier transforms (DFTs) are implemented with radix-2 fast Fourier transforms (FFTs).

Table 1: Complex baseband and analytic bandpass signal generation methods

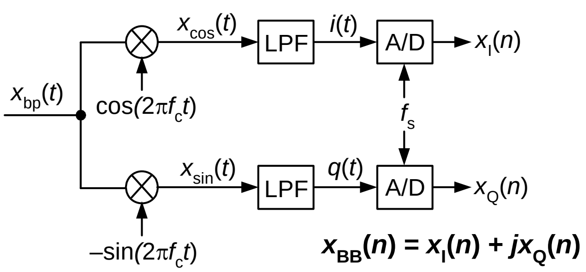
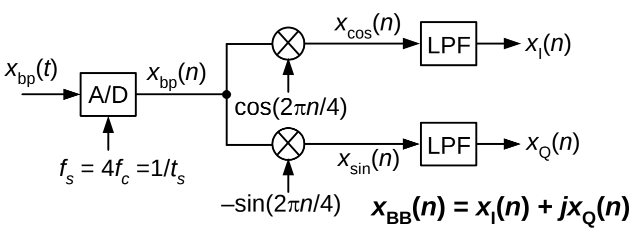
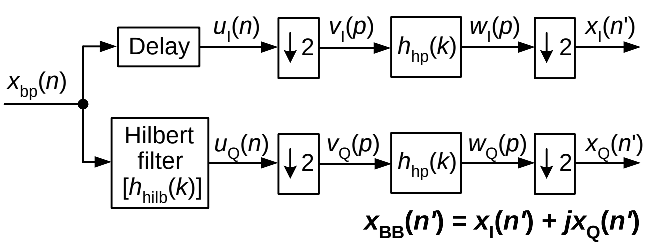
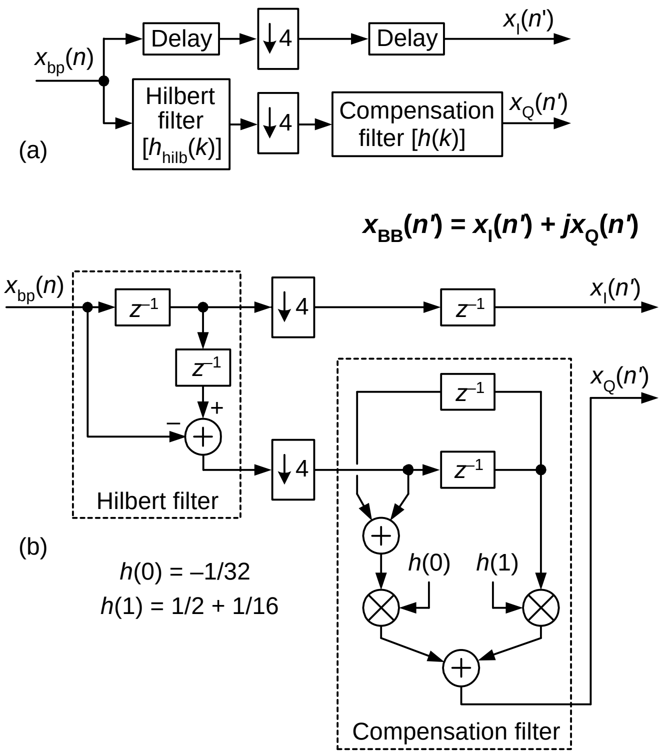
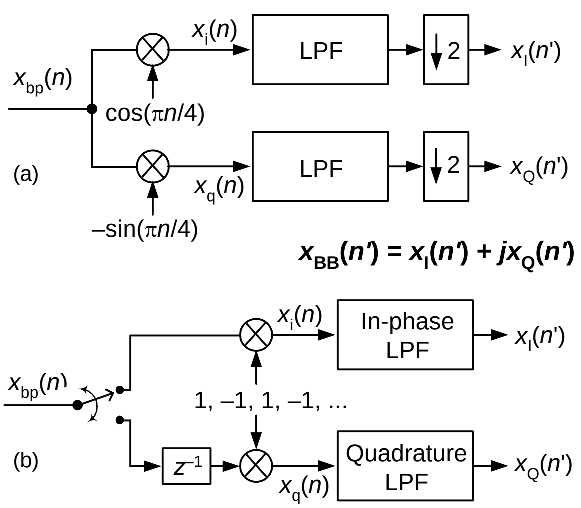
Process	Input/Output	Comments
Quadrature Sampling 	**Input:** analog bandpass signal centered at f_C Hz, with sample rate of f_S Hz.. **Output:** discrete complex $x_{BB}(n)$ baseband signal, centered at zero Hz, with sample rate of f_S Hz.	Uses analog mixing and analog lowpass filters. Difficult to control the exact phase delays and gains of the $i(t)$ and $q(t)$ signals.
Quadrature Sampling 	**Input:** analog bandpass signal centered at f_C Hz, with sample rate of f_S Hz. **Output:** discrete complex $x_{BB}(n)$ baseband signal, centered at zero Hz, with sample rate of f_S Hz.	All-digital downconversion facilitates exact control of phases and gains of the $x_I(n)$ and $x_Q(n)$ signals. A/D's f_S sample rate normally equal to $4f_C$, but setting $f_S = 0.8f_C$ allows bandpass sampling to reduce the f_S sample rate. f_S must greater than twice the bandwidth of $x_{bp}(t)$. See [1] or Section 8.9 of [2].
Discrete Complex Downconversion 	**Input:** discrete real bandpass signal centered at $f_S/4$ Hz, with sample rate of f_S Hz. **Output:** discrete complex $x_{BB}(n)$ baseband signal, centered at zero Hz, with sample rate of $f_S/4$ Hz.	Uses a time-domain Hilbert transformer and a half-band highpass FIR filter. If $h_{hilb}(k)$ has an odd number of taps, then half its coefficients will be zero-valued. See [3] or Section 13.43 of [2].

Table 1 Cont'd:

<table>
<tr>
<td>Discrete Complex Downconversion
</td>
<td>Input: discrete real bandpass signal centered at $f_S/4$ Hz, with sample rate of f_S Hz.

Output: discrete complex $x_{BB}(n)$ baseband signal, centered at zero Hz, with sample rate of $f_S/4$ Hz.</td>
<td>Uses a time-domain Hilbert transformer and simple three-tap highpass FIR compensation filter as shown in Part (a) of figure. Efficient implementation shown in Part (b) of figure. For reasonably acceptable operation, the bandwidth of $x_{bp}(n)$ must not be larger than, say, $f_S/10$. See Section 13.43 of [2].</td>
</tr>
<tr>
<td>Discrete Complex Downconversion
</td>
<td>Input: discrete real bandpass signal centered at $f_S/4$ Hz, with sample rate of f_S Hz..

Output: discrete complex $x_{BB}(n)$ baseband signal, centered at zero Hz, with sample rate of $f_S/2$ Hz.</td>
<td>Standard complex downconversion and lowpass FIR filtering, with decimation by two as shown in Part (a) of figure. Due to the decimation, a very efficient implementation is that shown in Part (b) of figure. The coefficients of the In-phase and Quadrature phase LPFs are decimated versions of the coefficients in the identical Part (a) LPFs. If the Part (a) LPFs are half-band filters, the Part (b) In-phase and Quadrature phase LPFs will be even more computationally efficient.
See [4], [5], or Section 13.1.3 of [2].</td>
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Table 1 Cont'd:

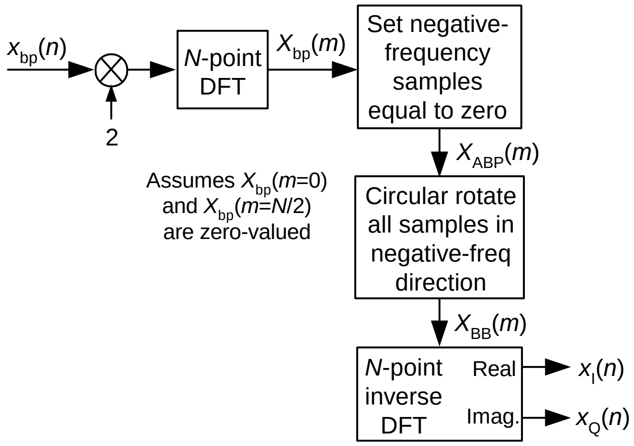
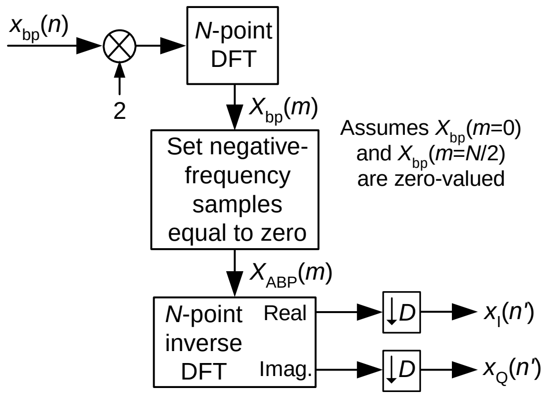
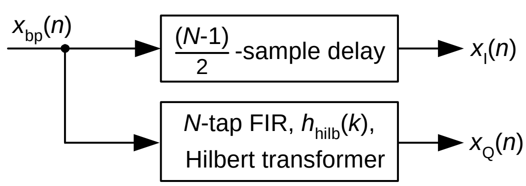
Discrete Complex Downconversion  $x_{BB}(n) = x_I(n) + jx_Q(n)$	**Input:** discrete real bandpass signal centered at f_C Hz, with sample rate of f_S Hz. **Output:** discrete complex $x_{BB}(n)$ baseband signal, centered at zero Hz, with sample rate of f_S Hz.	Negative-frequency components of $x_{bp}(n)$ are attenuated in the frequency domain. Frequency downconversion performed by shifting (rotating) the frequency-domain indices of the positive-frequency spectral samples prior to inverse DFT.
Discrete Complex Downconversion  $x_{BB}(n') = x_I(n') + jx_Q(n')$	**Input:** discrete real bandpass signal centered at kf_S/D Hz, with sample rate of f_S Hz. **Output:** discrete complex $x_{BB}(n)$ baseband signal, centered at zero Hz, with sample rate of f_S/D Hz.	Negative-frequency components of $x_{bp}(n)$ are attenuated in the frequency domain. Frequency downconversion performed by decimation in the time domain. See Section 13.29 of [2].
Analytic Bandpass Signal Generation  $x_{ABP}(n) = x_I(n) + jx_Q(n)$	**Input:** discrete bandpass signal centered at f_C Hz, with sample rate of f_S Hz. **Output:** discrete analytic $x_{ABP}(n)$ bandpass signal, centered at f_c Hz, with sample rate of f_S Hz.	Standard time-domain, tapped-delay line, FIR Hilbert transform method of discrete analytic bandpass signal generation. For proper time synchronization of $x_I(n)$ and $x_Q(n)$, N must be an odd number—in which case half the $h_{hilb}(k)$ coefficients will be zero-valued. See Note 61 of [7] or Section 9.4.1 of [2].

Table 1 Cont'd:

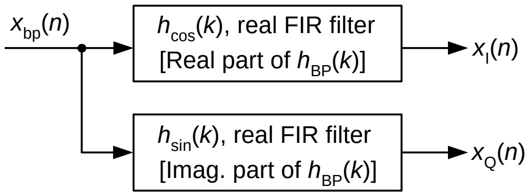
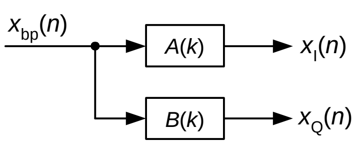
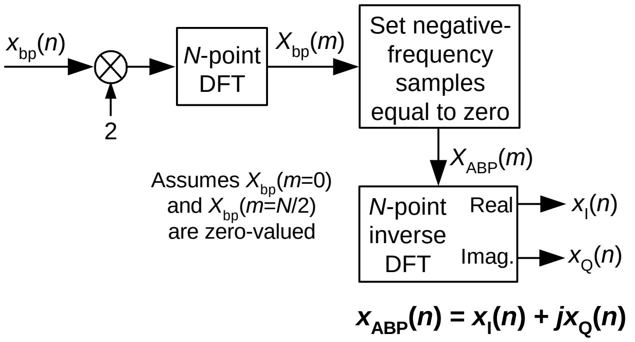
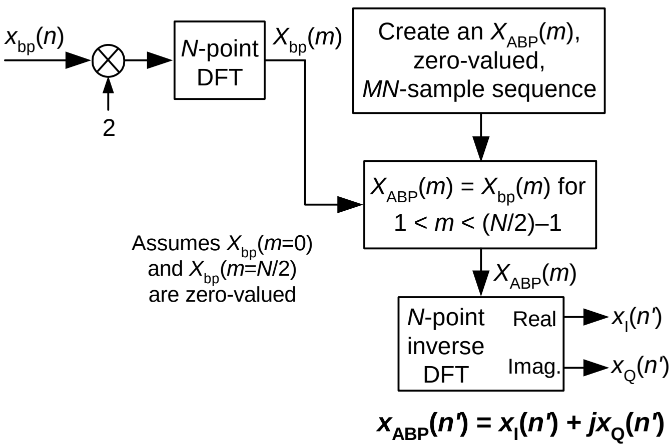
Analytic Bandpass Signal Generation  $\boldsymbol{x_{ABP}(n) = x_I(n) + jx_Q(n)}$ Windowing the two sets of filter coefficients will minimize passband magnitude differences between the $h_{cos}(k)$ and $h_{sin}(k)$ filters. (This method has slightly improved aggregate (overall) negative frequency attenuation compared to the following $A(k)$ and $B(k)$ dual filter method.)	**Input:** discrete bandpass signal centered at f_C Hz, with sample rate of f_S Hz. **Output:** discrete analytic $x_{ABP}(n)$ bandpass signal, centered at f_C Hz, with sample rate of f_S Hz.	A prototype lowpass filter (LPF) is designed to have a two-sided bandwidth slightly greater than the bandwidth of $x_{bp}(n)$. The LPF's coefficients are multiplied by cosine and sine sequences whose frequencies are f_C Hz, creating a positive-frequency complex $h_{bp}(k)$ bandpass filter. If $f_C = f_S/4$ then half the LPF coefficients will be zero-valued. Using a half-band LPF, if possible, further enhances computational efficiency. See [6], Note 62 of [7], or Section 9.5 of [2].
Analytic Bandpass Signal Generation  $\boldsymbol{x_{ABP}(n) = x_I(n) + jx_Q(n)}$ The $A(k)$ and $B(k)$ filters have guaranteed equal magnitude responses. $B[k]$ coefficients are a reversed-ordered version of the $A[k]$ coefficients (reduces coefficient storage requirement).	**Input:** discrete bandpass signal centered at f_C Hz, with sample rate of f_S Hz. **Output:** discrete analytic $x_{ABP}(n)$ bandpass signal, centered at f_C Hz, with sample rate of f_S Hz.	Clay Turner's method of designing two orthogonal real bandpass filters, that when combined yield a positive-frequency complex filter. If filters are centered at $f_S/4$ and have an even number of taps, then half the $A[k]$ and $B[k]$ coefficients will be zero-valued. Phase responses of $A[k]$ and $B[k]$ filters are very nearly, but not quite exactly, linear. See [8] for equations used to compute $A(k)$ and $B(k)$ coefficients.

Table 1 Cont'd:

Analytic Bandpass Signal Generation  	**Input:** discrete bandpass signal centered at f_C Hz, with sample rate of f_S Hz. **Output:** discrete analytic $x_{ABP}(n)$ bandpass signal, centered at f_C Hz, with sample rate of f_S Hz.	Straightforward frequency-domain method of bandpass analytic signal generation. Negative-frequency spectral components of $X_{bp}(m)$ are set to zero creating the desired analytic signal's spectrum. See [9] or Section 9.4.2 of [2].
Interpolated Analytic Bandpass Signal Generation  	**Input:** discrete bandpass signal centered at f_C Hz, with sample rate of f_S Hz. **Output:** interpolated discrete $x_{ABP}(n)$ analytic bandpass signal, centered at f_C Hz, with sample rate of Mf_S Hz.	Frequency-domain method of interpolated by *M* bandpass analytic signal generation. Negative-frequency spectral components of $X_{bp}(m)$ are set to zero creating the desired analytic signal's spectrum. New spectrum is zero-stuffed, prior to inverse DFT, to achieve time-domain interpolation by factor *M*. See Section 13.28.2 of [2].

For completeness, I mention that DSP pioneer Charles Rader proposed a computationally-efficient analytic bandpass signal generation method (See [10] or Note 66 in [7]) where both its $x_I(n)$ and $x_Q(n)$ output channels have identical frequency magnitude responses, however that scheme does not exhibit a linear-phase frequency response. As such, I didn't include it in Table 1.

References

[1] Considine, V. “Digital Complex Sampling,” *Electronics Letters*, 19, August 4, 1983.

[2] Lyons, R. *Understanding Digital Signal Processing*, Prentice Hall Publishing, Hoboken, NJ, 2010.

[3] Ohlsson, H., et al. “Design of a Digital Down Converter Using High Speed Digital Filters,”

in *Proc. Symp. on Gigahertz Electronics*, Gothenburg, Sweden, March 13–14, 2000, pp. 309–312.

[4] Powell, S. “Design and Implementation Issues of All Digital Broadband Modems,” *DSP World Workshop Proceedings*, Toronto, Canada, September 13–16, 1998, pp. 127–142.

[5] Frerking, M. *Digital Signal Processing in Communications Systems*, Chapman & Hall, New

York, 1994, p. 330.

[6] Reilly, A., et. al. “Analytic Signal Generation—Tips and Traps,” *IEEE Trans. on Signal Proc.*,

Vol. 42, No. 11, Nov. 1994.

[7] Rorabaugh, C. Notes on Digital Signal Processing: Practical Recipes for Design, Analysis and Implementation, Prentice Hall Publishing, Hoboken, NJ, 2010.

[8] Turner, C. "An Efficient Analytic Signal Generator", *IEEE Signal Processing Magazine, DSP Tips & Tricks column*, July 2009.

[9] Marple, S., Jr. “Computing the Discrete-Time ‘Analytic’ Signal via FFT,” *IEEE Trans. on*

Signal Proc., Vol. 47, No. 9, September 1999, pp. 2600–2603.

[10] Rader, C. “A Simple Method for Sampling In-Phase and Quadrature Components,” *IEEE Trans. Aerospace and Electronic Syst.*, Vol. 20, No. 6, pp. 821-824, Nov. 1984.